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OPTIMAL PURCHASE QUANTITY AND PERMISSIBLE CREDIT PERIOD FOR CRITICAL PRODUCT

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To boost market demand of a product and to attract more potential buyers, a reasonable credit period be offered by seller's/manufacturer's. The demand is considered to be increasing with time and depends upon the offer of the seller's credit period. A trade-off between demand and the credit period is to be settled. In this paper, we incorporate the risk incurred in sales revenue for profit maximisation. The necessary and sufficient conditions to settle a trade-off between optimal permissible credit period and purchase quantity be discussed.

Finally, the numerical solutions will validate the theoretical results.

KEYWORD : Inventory, Trade-off, Risk optimisation.

INTRODUCTION

To survive in perfect competitive business environment, the seller offers its potential buyers a credit period to settle the account which is against the quantity. Moreover, sellers did not impose an interest if buyer is ready to pay and settle their due account within the specified credit period with mutual understanding of sellers and buyers. This offer is beneficial in favour of seller's point of view.

(a) it attracts potential buyers

(b) it is considered to be replacement of price discount of product and does not compel sellers to negotiate on prices.

However, the delay in payment enhances the default risk. In literature EOQ model for the buyer, with a fixed credit period be offered by seller has been developed by Shah [4]. The review of literature emphasis on buyer's point of view for fixed credit period. Several researcher developed retailer's optimal policy with credit linked demand with payment in permissible delayed period. Jaggi [2], Lou and Wang [3] discussed the seller's decision about pre-specified delay in payment.

In this paper, we develop an EOQ model for the seller's to incorporate two significant facts:

(a) Trade credit dependant demand

(b) Risk due to offer of the trade credit

We establish the necessary condition for obtaining the optimality and the impact of various inventory parameters . The non-linearity of the objective function renders us to obtain a closed form solution to the seller's optimal credit period and quantity purchased. The model is validated with suitable numerical example.

ASSUMPTIONS

(a) Seller deals with single item

(b) Replenishment rate is infinity

(c) Shortages are not allowed

(d) Lead time is negligible

(e) Sellers keep selling price constant to retain his buyers

(f) Trade credit is similar to price discount [5] and consider demand rate to be function of credit period and time such that

$$R\,(M, t) = a\,(1 + bt)\,M^{\beta}$$

where $a > 0$ and $0 \le b < 1$.

Which is rate of change of demand with respect to time and $\beta > 0$, is a constant.

(g) From seller's point of view, risk increases with longer credit period, hence, the rate of risk for given credit period M can be assumed as

$$F\,(M) = 1 - M^{-\Upsilon};\ \Upsilon > 0 \text{ is a constant.}$$

NOTATIONS

The following notations are used in analysing the problem:

A = Ordering cost / order

C = Purchase cost / unit

P = Selling Price / Unit and $P > C$

M = Credit period offered, which is a decision variable.

$R\,(M, t)$ = Demand rate dependent on Time and credit period

$I\,(t)$ = Inventory level at any instant t, where $0 \le t \le T$

T = Cycle time, a decision variable

Q = Seller's purchase Quantity

$\Pi\,(M, t)$ = Seller's Profit per unit time

$S\,R$ = Seller's revenue after risk

MATHEMATICAL ANALYSIS

Rate of change of inventory can be analysed with the following differential equation :

$$\frac{dI\,(t)}{dt} = -\,R(M,t)$$

$$\frac{dI\,(t)}{dt} = -\,a\,(1+bt)M^{\beta} \qquad \text{....(1)}$$

With initial condition, $I(T) = 0$

The solution of equation (1) can be computed as :

$$\int dI(t) = \int_t^T a(1+bt)M^{\beta}\, dt$$

or $$I(t) = aM^{\beta}\left[t + \frac{1}{2}bt^2\right]_t^T$$

or $$I(t) = aM^{\beta}\left[(T-t) + \frac{1}{2}\, b(T^2 - t^2\right] \quad \text{... (2)}$$

And the initial inventory which is the Quantity $Q = I(0)$

$$= aM^{\beta}\left[(T-0) + \frac{1}{2}b(T^2 - 0)\right]$$

$$= aM^{\beta}\left(T + \frac{1}{2}bT^2\right) \quad \text{... (3)}$$

and relevant parameter from seller's point of view are respectively

(a) Revenue after risk

$$\text{SR} = P\int_0^T R(M, t)\, dt\ (1 - F(M))$$

$$= P\int_0^T a(1+bt)M^{\beta}\, dt\, M^{-\gamma}$$

$$= aPM^{\beta-\gamma}\int_0^T (1+bt)\, dt$$

$$= aPM^{\beta-\gamma}\left[t + \frac{1}{2}bt^2\right]_0^T$$

$$= aPM^{\beta-\gamma}\left[T + \frac{1}{2}bT^2\right]$$

(b) Purchase cost = $Q \,.\, C$

$$= a\,M^{\beta}\left(T + \frac{1}{2}bT^2\right).C$$

(c) Ordering Cost = A

(d) Holding Cost $= h\int_0^T I(t)\, dt$

$$= h\int_0^T a\,M^{\beta}\left[(T-t) + \frac{1}{2}b(T^2 - t^2)\right] dt$$

$$= ha\,M^{\beta}\left[Tt - \frac{t^2}{2} + \frac{1}{2}b\left(T^2 t - \frac{t^3}{3}\right)\right]_0^T$$

$$= ha\,M^{\beta}\left[T^2 - \frac{T^2}{2} + \frac{1}{2}b\left(T^3 - \frac{T^3}{3}\right) - 0\right]$$

$$= ha\,M^{\beta}\left[\frac{T^2}{2} + \frac{bT^3}{3}\right]$$

$$= ha\,M^{\beta}T^2\,\frac{3+2bT}{6}$$

and consequently the seller's profit per unit time is equal to

$\Pi\,(M, t) = 1/T\,(SR\text{-}QC\text{-}A\text{-}$Holding Cost)

$\Pi\,(M, t) = 1/T\,(P\int_0^T R\,(M,t)dt\,(1-F(M)) - QC - A - h\int_0^T I(t)dt\,)$ … (4)

The necessary condition per credit period and cycle time for maximizing annual profit per unit time, we have

$$\frac{\partial\pi(M,T)}{\partial M} = 0$$

or $\dfrac{\partial}{\partial M}\left[\dfrac{1}{T}\left\{aPM^{\beta-\gamma}\left(T+\dfrac{1}{2}bT^2\right) - aM^{\beta}\left(T+\dfrac{1}{2}bT^2\right)C - A - haM^{\beta}T^2\,\dfrac{3+2bT}{6}\right\}\right] = 0$

or $\dfrac{\partial}{\partial M}\left[aPM^{\beta-\gamma}\left(1+\dfrac{1}{2}bT\right) - aM^{\beta}\left(1+\dfrac{1}{2}bT\right)C - \dfrac{A}{T} - haM^{\beta}T\,\dfrac{3+2bT}{6}\right] = 0$

or $aP(\beta-\gamma)M^{\beta-\gamma-1}\left(1+\dfrac{1}{2}bT\right) - a\beta M^{\beta-1}\left(1+\dfrac{1}{2}bT\right)C - 0 - ha\beta M^{\beta-1}T\,\dfrac{3+2bT}{6} = 0$

or $\dfrac{a}{6M}$ $[6P\,(\beta-\gamma)\,M^{\beta-\gamma} + 3b\,P\,(\beta-\gamma)\,M^{\beta-\gamma}.T - 6\beta\,\mathrm{M}^{\beta}C - 3\beta bCM^{\beta}T - 3h\beta\,M^{\beta}$
$- 2h\,\beta\,b\,M^{\beta}T] = 0$

or $\dfrac{a}{6M}$ $[-3Pb\,\beta\,M^{\beta-\gamma}T - 6P\,\beta\,M^{\beta-\gamma} + 3Pb\,\gamma\,M^{\beta-\gamma}T + 6P\,\gamma\,M^{\beta-\gamma} + 6C\,\beta\,M^{\beta}$
$+ 3Cb\beta\,M^{\beta}T + 2hP\beta\,b\,M^{\beta}T^2 + 3h\beta\,M^{\beta}T] = 0$

and $\dfrac{\partial\pi\,(M,T)}{\partial T} = 0$

or $\dfrac{\partial}{\partial T}\left[aPM^{\beta-\gamma}\left(1+\dfrac{1}{2}bT\right) - aM^{\beta}\left(1+\dfrac{1}{2}bT\right)C - \dfrac{A}{T} - ahM^{\beta}\,\dfrac{3T+2bT^2}{6}\right] = 0$

or $aPM^{\beta-\gamma}\left(\dfrac{b}{2}\right) - aM^{\beta}\left(\dfrac{b}{2}\right).C - A\left(-\dfrac{1}{T^2}\right) - ahM^{\beta}\,\dfrac{3+2b.2T}{6} = 0$

or $\dfrac{1}{2}aPM^{\beta-\gamma}b - \dfrac{1}{2}aM^{\beta}bC + \dfrac{A}{T^2} - \dfrac{3}{6}ahM^{\beta} - \dfrac{4}{6}ahbM^{\beta}T = 0$

or $\dfrac{1}{6T^2}[3aPbM^{\beta-\gamma} - 3abCM^{\beta} + 6A - 3ahM^{\beta}T^2 - 4ahbM^{\beta}T^3] = 0$

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Which is the explicit or closed form of solution with known parameters.

NUMERICAL EXAMPLE

For A = ₹ 85 per order

C = ₹ 9 per unit

H = ₹ 3.75 per unit per annum

a = 900 units, b= 45%, $\beta = 4$, $\Upsilon = 2$

P = ₹ 13 per unit

Now, for maximizing annual profit per unit time, the credit period M for different cycle time may be calculated as follows:

(a) For Cycle Time (T) = 0.5, we have,

Credit Period (M) = 0.8492,

Seller's Purchase Quantity (Q) = 260.2454 Units and

Seller's Profit per Unit Time π (M, T) = ₹ 4522.7987

(b) For T = 1, we have,

M = 0.8495,

Q = 574.5565 units and

π (M, T) = ₹ 5082.5742

(c) For T = 1.5, we have,

M = 0.8495

Q = 940.9827 units and

π (M, T) = ₹ 5562.18395

Thus, we observe that there is no major change in credit period with different cycle time, but purchase quantity and profit are increasing with increasing cycle time, which is beneficial from seller's point of view.

The behaviour of demand with time and delay period is shown in figures as follows for different cycle time:

GRAPHICAL REPRESENTATION

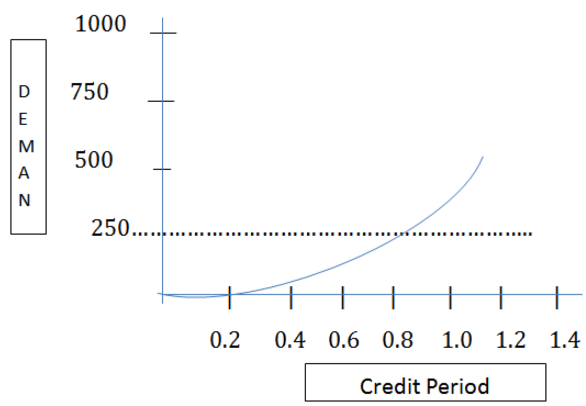

Fig. (a)

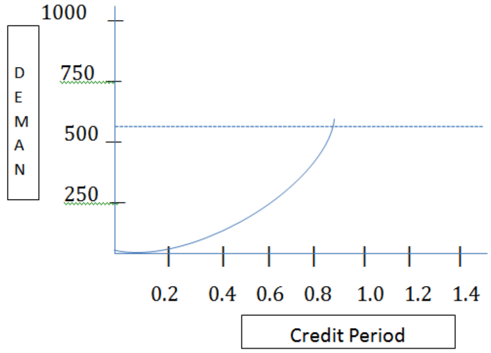

Fig.(b)

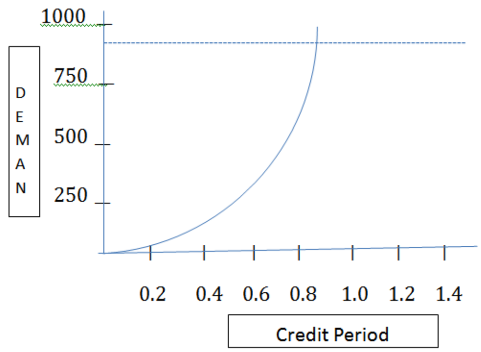

Fig.(c)

CONCLUSION

It is difficult to set a permissible credit period from seller's point of view for trade-off an item, whose demand exists in the perfect market. We observe that the problem become more realistic and challenging, when demand is increasing linearly with time. Further, we observe that higher values of demand and its selling price suggest seller to provide more flexible credit period and thereby receive more order for a specific period. On the basis of the trade-off exist between seller's and buyer's setting of optimal credit period from seller's point of view are significant. Numerical examples validate the optimal credit period.

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